Int. J. Heat Mass Transfer Vol. 28, No. 4, pp. 884–885, 1985
Printed in Great Britain

0017 9310/85 $3.00 + 0.00
Pergamon Press Ltd

Simplified relations for the phase change process in spherical geometry

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(*Received* 30 *April* 1984)

NOMENCLATURE

c	specific heat of solid material
h	heat transfer coefficient
κ	thermal conductivity of solid material
L	latent heat of fusion
r	radial position
r_f	radius of solidification front
R	radius of sphere
S	solidified thickness for the slab
t	time
T	temperature
T_f	temperature of fusion
T_∞	temperature of cooling fluid
α	thermal diffusivity of solid material
ρ	density.

INTRODUCTION

EXACT SOLUTIONS to the problem of heat transfer with phase change are only known for the semi-infinite slab. The object of the present work is to develop a correction factor which when applied to the exact solution of the semi-infinite slab will give a solution valid for the sphere. Exact solutions obtained for the slab are compared with numerical solutions for the sphere under the same conditions and a correction factor is obtained.

DESCRIPTION OF THE PROBLEM

According to the virtual system approach [1] for the slab, the solidification time t in terms of the solidified thickness S is, for Newtonian cooling without superheating,

$$t = \frac{S^2}{4\alpha\phi^2} + \frac{\rho L S}{h(T_f - T_\infty)} \tag{1}$$

where the solidification constant ϕ is defined as

$$\sqrt{\pi}\phi \, \mathrm{erf}(\phi) \exp(\phi^2) = Ste. \tag{2}$$

Considering the following dimensionless parameters

$$\tau = \frac{\alpha t}{R^2} \qquad \varepsilon = \frac{S}{R} = 1 - \frac{r_f}{R}$$
$$Bi = \frac{hR}{K} \qquad Ste = \frac{c(T_f - T_\infty)}{L} \tag{3}$$

we may write (1) in the form

$$\tau = \frac{\varepsilon^2}{4\phi^2} + \frac{\varepsilon}{Ste \cdot Bi}. \tag{4}$$

In order to obtain the correction factor to be applied to equation (4) to make it valid for spheres, it was initially solved for Stefan numbers ranging from 0.1 to 3.0 and for Biot numbers from 1.0 to infinity for each Stefan number. The next step was to evaluate numerically [2] for the sphere the results corresponding to the same combination of Stefan and Biot numbers. Table 1 shows the results of the ratio τ_s/τ_p for $\varepsilon = 1$ where τ_s is the dimensionless solidification time for the sphere numerically obtained and τ_p is the dimensionless solidification time according to equation (4).

As it can be seen from Table 1, the relation between the

Table 1. Ratio between the total solidification time for the sphere and for the plane

		Stefan No.				
		0.1	0.5	1.0	2.0	3.0
Biot No.	1.0	0.36	0.41	0.45	0.51	0.56
	2.0	0.36	0.41	0.46	0.52	0.56
	5.0	0.36	0.41	0.46	0.51	0.55
	10.0	0.36	0.40	0.45	0.50	0.53
	∞	0.36	0.41	0.45	0.51	0.53

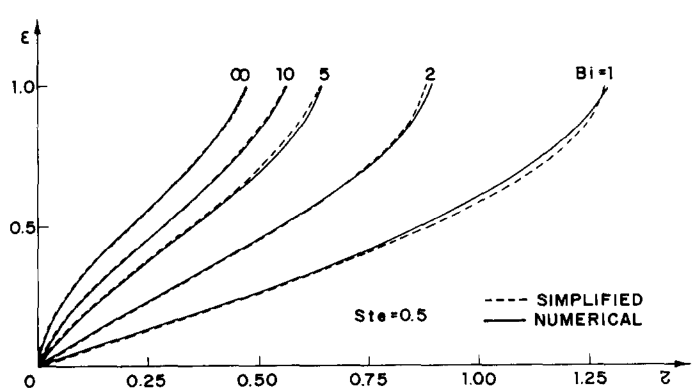

FIG. 1. Position of sphere interface as function of time.

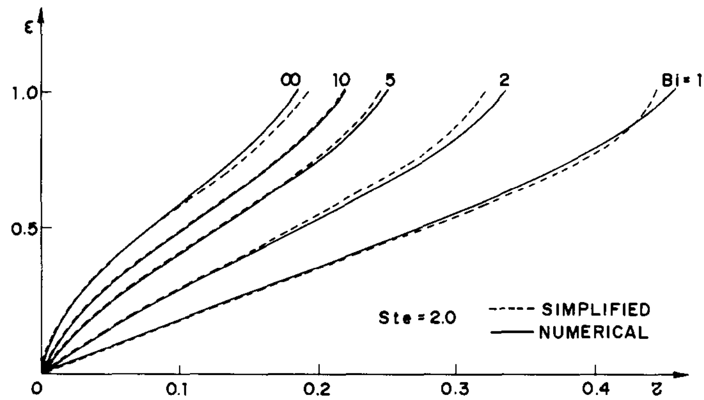

FIG. 2. Position of sphere interface as function of time.

solidification times increases with the Stefan number. However, we notice that this relation varies very little with the Biot number for a specific Stefan number. This fact suggests that the correction factor to be obtained can be a function only of the Stefan number, independently of the Biot number. The best expression found for the correction factor was of the polynomial type:

$$\tau_s = (1+a\varepsilon+b\varepsilon^2)\cdot\left[\frac{\varepsilon^2}{4\phi^2}+\frac{\varepsilon}{Ste\cdot Bi}\right]. \quad (5)$$

The coefficients a and b were determined by considering the solidification time for the sphere and the plane from $\varepsilon = 0$ to $\varepsilon = 1$ for regular intervals of $\Delta\varepsilon = 0.05$. The results of both geometries for a given Stefan number and all the Biot numbers considered were compared and related by means of the least square technique. Thus, a and b were found to be

$$a = -0.803+0.309\,(Ste)-0.036\,(Ste)^2$$
$$b = 0.160-0.210\,(Ste)+0.022\,(Ste)^2. \quad (6)$$

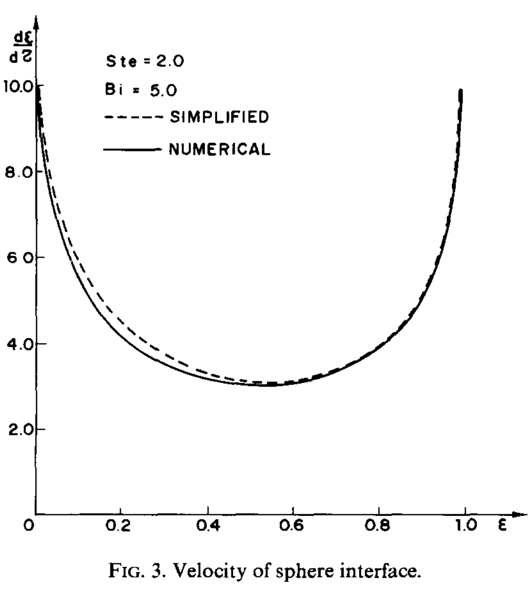

FIG. 3. Velocity of sphere interface.

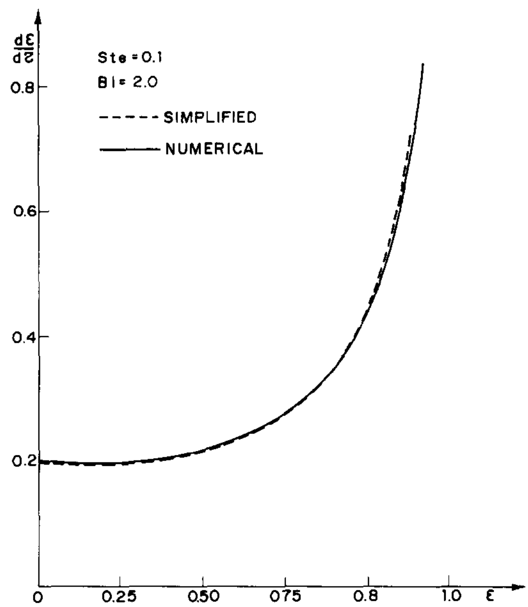

FIG. 4. Velocity of sphere interface.

The velocity of the solidification front is given by

$$\frac{d\varepsilon}{d\tau} = \left[\frac{\varepsilon}{4\phi^2}(2+3a\varepsilon+4b\varepsilon^2)+\frac{1}{Ste\cdot Bi}(1+2a\varepsilon+3b\varepsilon^2)\right]^{-1}. \quad (7)$$

The percentage absolute mean error of the solidification time calculated according to this corrected method is less than 5% compared to the numerical results.

Figures 1 and 2 show the solidification front position as function of time and Figs. 3 and 4 show the solidification front velocity as function of its position.

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